If $D-u_0<0$, the undisturbed and compressed regions are interchanged and we have a « backward-facing shock wave ». These two cases are illustrated in Fig. 1.

Material is always accelerated in the direction of propagation of the shock wave. From eqs. (7)-(9) the change in mass flow or particle velocity can be calculated:

$$u_1-u_0=\pm[(p_1-p_0)(V_0-V_1)]^{\frac{1}{2}}. \tag{10}$$

If $D-u_0>0$, $u_1-u_0>0$; if $D-u_0<0$, $u_1-u_0<0$.

Equation (9) contains the thermodynamics of the shock transition and is called the « Rankine-Hugoniot equation ». For « normal » materials it can be satisfied only by compressive waves, $p_1>p_0$. We arbitrarily define « normal » materials as those for which the adiabatic (p, V) relation in one-dimensional compression, sometimes called « uniaxial strain », is concave upward. Most fluids are normal in this sense, and most solids are normal over a restricted range. For such materials the limitation of shock waves to compressive waves follows from both hydrodynamic and thermodynamic considerations. As to the latter, we find that by differentiating eq. (9) and combining it with the First and Second Laws of Thermodynamics an expression is obtained for entropy change along the Hugoniot:

$$\mathrm{d}S_1/\mathrm{d}p=[(V_0-V_1)/2T_1]\left[1-\frac{(p_1-p_0)/(V_0-V_1)}{|\mathrm{d}p/\mathrm{d}V|_{V_1}}\right], \tag{11}$$

where T_1 is temperature at (p_1, V_1). The bracket in eq. (11) is positive for all $p_1>p_0$ if the Hugoniot is concave upward. This is illustrated in Fig. 2, where it is obvious that the slope of the chord from 0 to A is less in magnitude than the slope of the tangent at A.

The increase in entropy in the shock front is produced by the presence of dynamic or irreversible forces associated with viscosity, stress-relaxation and the like. These forces are responsible for maintaining the linear relation between p and V in eq. (5): at any point of compression the total compressive stress p in the x-direction is the sum of an equilibrium and of a dynamic contribution. A physically unreal but mathematically interesting problem is to let the dynamic stress vanish and to represent the equilibrium stress by an equation of state,

$$p=\bar{p}(V, E).$$

In the absence of other irreversible processes, such as heat conduction, the shock front then becomes a mathematical discontinuity connecting states (p_0, V_0, u_0, E_0) and (p_1, V_1, u_1, E_1) [1].

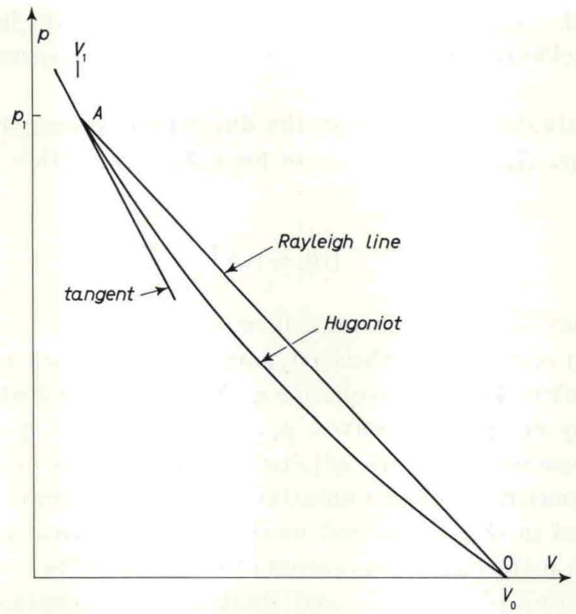

Fig. 2. – Entropy increases with pressure for a normal material.

A more realistic approximation is to add a viscous stress:

$$p = \overline{p}(V, E) - \alpha \, \mathrm{d}V/\mathrm{d}t \, . \tag{12}$$

If the dependence on E can be neglected, eqs. (5) and (12) together become a differential equation for the density profile in the shock front:

$$\alpha D \mathrm{d}V/\mathrm{d}\xi = p_0 + m^2(V_0 - V) - \overline{p}(V) \, . \tag{13}$$

In case the E-dependence of eq. (12) cannot be neglected, eqs. (5), (6) and (12) provide a description of the profile.

The sign of the entropy change in eq. (11) is directly related to the propagation process. For the shock wave of Fig. 1, moving into stationary material ($u_0 = 0$):

$$D^2 = V_0^2(p_1 - p_0)/(V_0 - V_1) > V_0^2 |\mathrm{d}p/\mathrm{d}V|_{V_0} \equiv c_0^2 \tag{14}$$

provided $(\partial^2 p/\partial V^2)_S > 0$ at (p_0, V_0). The velocity of sound is defined as $c = V(-\partial p/\partial V)_S^{\frac{1}{2}}$. The identity between $(\partial p/\partial V)_S$ and $\mathrm{d}p/\mathrm{d}V$ on the Hugoniot at (p_0, V_0) is possible because isentrope and Hugoniot have a second-order contact at the foot of the Hugoniot. This will be discussed later by ROYCE and KEELER. The inequality in (14) tells us that the shock wave overtakes any acoustic wave ahead of it. Under the same conditions a disturbance behind the shock front overtakes the shock, provided, of course, that both